A Utility-based Distributed Maximum Lifetime Routing Algorithm for Wireless Networks

Yuan Xue, Yi Cui, Klara Nahrstedt
Department of Computer Science
University of Illinois at Urbana-Champaign
{*xue, yicui, klara*}@cs.uiuc.edu

***Abstract*— Energy efficient routing is a critical problem in multihop wireless networks due to the severe power constraint of wireless nodes. Despite its importance and many research efforts towards it, a distributed routing algorithm that maximizes network lifetime is still missing. To address this problem, we propose a novel utility-based nonlinear optimization formulation to the maximum lifetime routing problem. Based on this formulation, we further present a fully distributed, localized routing algorithm, which is proved to converge to the optimal point, where the network lifetime is maximized. Solid theoretical analysis and simulation results are presented to validate our solution.**

I. Introduction

Multihop wireless network can be formed by wireless nodes with no pre-existing and fixed infrastructures. In order to provide communication throughout the network, the wireless nodes cooperate to handle network functions, such as packet routing. One example of multihop wireless network is sensor network, which can be readily deployed in diverse environments, such as health care, military, and disaster detection, to collect and process useful information in a autonomous manner.

One of the most important issues in wireless network is the energy constraint – wireless nodes carry limited, irreplaceable, power supply. Moreover, radio communication consumes a large fraction of this supply. Such observations pose critical demand to design *energy efficient* packet routing algorithms. For such algorithms to scale to larger networks, localized algorithms need to be proposed. The key design challenge is to derive the desired global system properties in terms of energy efficiency from the localized algorithms.

In the existing works, the problem of designing energy-efficient routing algorithms has been extensively studied in both general multihop wireless networks [1], [2], [3], [4], [5], and the particular backdrop of sensor networks [6], [7], [8], [9], [10]. Various goals may be achieved by these energy-efficient routing algorithms, such as minimizing energy consumption for end-to-end paths [11], [4], [12], [13], [14], [15], [16], or maximizing the lifetime of the whole network [1], [2], [3], [6], [17]. Here we give a brief overview of these existing approaches from an *optimization theoretical* point of view, and highlight the original contribution of this work in light of previous works.

A. Minimum energy routing: User optimization

Minimum energy routing problem presents a “user optimization” problem. It tries to optimize the performance of a single user (an end-to end connection), minimizing its energy consumption. To solve this problem, the typical approach [14], [15] is to use a shortest path algorithm in which the edge cost is the power consumed to transmit a packet between two nodes of this edge. Though effectively reducing the energy consumption rate, this approach can cause unbalanced consumption distribution, *i.e.,* the nodes on the minimum-energy path are quickly drained of energy, causing network partition or malfunctioning. Some routing algorithms [4], [12] associate a cost with the node of low energy reserve, but they remain a heuristic solution.

B. Maximum lifetime routing: System optimization

Maximum network lifetime routing problem tries to maximally prolong the duration in which the entire network properly functions. It presents a “system optimization” problem, which is radically different from “user optimization”. To achieve the goal of “system optimization”, global coordination is required, which poses significant challenge to the design of a distributed routing algorithm. On the other hand, maximum lifetime routing well addresses the power consumption balance problem of the minimum energy routing and studies a more critical issue of wireless networks.

1) Existing approach – Linear optimization formulation: The inherit characteristic of maximum lifetime routing problem naturally leads to a *linear optimization* formulation [2], [3], [17], [1], [6]. In this formulation, the problem is usually considered as a multicommodity problem. *Combinatorial* algorithm can then be designed to solve this problem. In particular, the following algorithms have been presented in the existing work.

- *Heuristic*. Heuristic algorithms are presented in the work of [2] and [6]. Being heuristics, these algorithms lack solid theoretical proof of their performance. For example, the algorithm in [2] may have degraded worst case performance. And the algorithm in [6] does not scale well to the network size in terms of running time.
- *Centralized approximation algorithm*. In [3], Chang and Tassiulas adopts a classical linear optimization approach – the Garg-Koenemann [18] algorithm – for multicommodity flow and provide a centralized combinatorial

approximation solution. Being a centralized algorithm, it is hard to be deployed on realistic wireless network environments.

- *Distributed combinatorial algorithm*. In a recent work, Sankar and Liu [1] adopt a distributed flow algorithm due to Awerbuch and Leighton [19], [20]. Yet, the core of this algorithm can only *verify whether a traffic input can be satisfied by a required network lifetime, and if so, how to route traffic*. It does *not directly answer the question, given traffic input what is the maximum network lifetime, and how to achieve it via routing*. To calculate the exact network lifetime, a bisection search is needed. Thus to deploy this algorithm for routing packets online, it will suffer from slow convergence and potential performance fluctuation due to the bisection search.

2) Our Approach – Nonlinear optimization: In this paper, we aim at designing a fully distributed routing algorithm that achieves the goal of maximizing network lifetime. Towards this goal, we propose a utility-based nonlinear optimization formulation of the maximum lifetime routing problem. The essential idea of this formulation is based on the following observation. The goal of maximum network lifetime is to maximize the lifetime of the node who has the minimum lifetime among all nodes. If we regard lifetime as a "resource", then this goal can be regarded as to "allocate lifetime" to each node so that max-min fairness criterion is satisfied. This "lifetime allocation" mechanism needs to be achieved via routing and has to satisfy the traffic demand constraint. From this view, we further adopt the concept of "utility" which has been widely used in resource allocation in economics as well as distributed computing. By defining an appropriate utility function based on lifetime, the problem of achieving max-min lifetime allocation is converted into aggregated utility maximization problem.

Based on this formulation, the key to the distributed algorithm is to consider the marginal utility at each node. We show that the system achieves optimal routing when the marginal utilities of all nodes are equal. Hence the design philosophy of a distributed routing algorithm is to let each node balance its own marginal utility by iteratively adjusting traffic on different routing paths. Such a design philosophy was first proposed by Gallager [21]. The problem studied in that work is to minimize the overall network delay through distributed routing. Gallager's algorithm was later improved by Bertsekas et al. [22]. Although sharing the same design thought, our problem has entirely different optimization goal with different objective function.

To summarize, despite the importance of the maximum network lifetime problem and many research efforts towards it, a fully distributed, localized on-line routing algorithm is still missing. Towards this goal, this paper makes the following original contributions. First, this paper presents a unique utility-based nonlinear optimization formulation of the maximum lifetime routing problem. Second, based on this formulation, this paper presents a fully distributed, localized routing algorithm to solve this problem. It also presents solid theoretical analysis of the properties of this algorithm. To the best of our knowledge, this is the first localized maximum lifetime routing algorithm.

It is important to note that, although designed for maximum lifetime routing in wireless networks, the presented algorithm can also be extended to other scenarios where a global optimization goal needs to be achieved by distributed routing.

The rest of this paper is organized as follows. We model the network in Sec. II, and present the utility-based formulation of the network lifetime maximization problem in Sec. III. Sec. IV and Sec. V present the optimality condition of the solution and the distributed maximum network lifetime routing algorithm and its analysis. Sec. VI shows the performance study, Sec. VII presents the related work, and Sec. VIII concludes the paper.

II. Model

We consider a multihop wireless network which consists of a set of wireless nodes, represented as $\mathcal{N} = \{1, 2, \ldots, n\}$. Two nodes that are within the transmission range of each other can communicate directly and form a wireless link. Let $\mathcal{L}$ be the set of wireless links, denoted as $\mathcal{L} = \{(i,k) \mid \text{an wireless link goes from } i \text{ to } k\}$. Each link (i,k) has a weight d_{ik}, which is the distance between the antennas of node i and k.

To illustrate how traffic is routed in the network, we further introduce the following definitions and notations.

- *input traffic* $r_i(j) \geq 0$ is the traffic (in bit per second), *generating* at node i and destined for node j.
- *node flow* $t_i(j)$ is the *total* traffic at node i destined for node j. $t_i(j)$ includes both $r_i(j)$ and the traffic from other nodes that is routed through i to destination j.
- *routing variable* $\phi_{ik}(j)$ is the *fraction* of the node flow $t_i(j)$ routed over link (i,k). This variable defines a routing solution. It is obvious that
  - $\phi_{ik}(j) = 0$, if $(i,k) \notin \mathcal{L}$, as no traffic can be routed through non-existent link;
  - $\phi_{ik}(j) = 0$, if $i = j$, because traffic that has reached its destination is not sent back into the network;
  - as node i must route its entire node flow $t_i(j)$ through all outgoing links,

$$\sum_{k \in \mathcal{N}} \phi_{ik}(j) = 1, \forall i, j \in \mathcal{N}$$

We illustrate above concept in Fig. 1. In the figure, the input traffic is $r_2(4) = 2$Kbps, $r_3(6) = 3$Kpbs, and $r_1(6) = 1$Kbps, respectively. The node flows are $t_2(4) = r_2(4)$, $t_3(6) = r_3(6)$, $t_1(4) = 2$Kbps, $t_1(6) = 4$Kbps, $t_4(6) = 1$Kbps, and $t_5(6) = 3$Kbps. The routing variables are as follows: $\phi_{21}(4) = \phi_{31}(6) = \phi_{14}(4) = \phi_{46}(6) = \phi_{56}(6) = 1$, $\phi_{14}(6) = 1/4$, and $\phi_{15}(6) = 3/4$.

One important constraint of traffic routing in a network is *flow conservation*: the traffic into a node for a given destination is equal to the traffic out of it for the same destination. This constraint can be formally expressed as follows.

$$t_i(j) = r_i(j) + \sum_{l \in \mathcal{N}} t_l(j) \phi_{li}(j), \forall i, j \in \mathcal{N} \tag{1}$$

We can also understand the routing problem as a multicommodity problem, i.e., the flow to each destination node j is

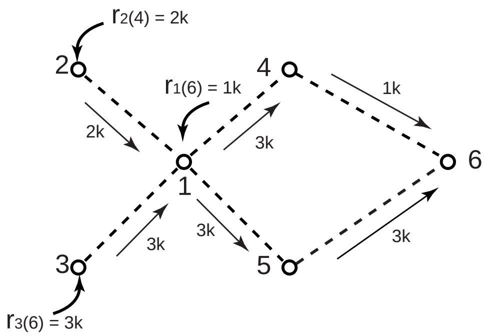


Fig. 1. Example network and traffic

regarded as a commodity j. Then the above statement simply means that the amounts of a commodity that enters and leaves a node must be the same.

Now we present the energy model used in this paper. We define E_i the energy reserve at node i. Let p_i^r (J/bit) be the power consumption at node i, when it receives one unit of data, and p_{ik}^t (J/bit) be the power consumption when one unit of data is sent from i over link (i,k). Based on the first order radio model, we have the following.

$$p_i^r = \alpha \tag{2}$$
$$p_{ik}^t = \alpha + \beta \cdot d_{ik}^m \tag{3}$$

where α is a distance-independent constant that represents the energy consumption to run the transmitter or receiver circuitry, and β is the coefficient of the distance-dependent term that represents the transmit amplifier. The exponent m is determined from field measurements, which is typically a constant between 2 and 4.

The routing variables $\phi_{ik}(j), \forall j$ and the node flow set $t_i(j), \forall j$ jointly determine the traffic sent along wireless link (i,k). Further, $t_i(j), \forall j$ and the input traffic $r_i(j), \forall j$ jointly determine the traffic received at node i. Thus we have the wireless node i's power consumption rate p_i, in J/sec as follows:

$$p_i = \sum_{j \in \mathcal{N}} \Big[t_i(j) \sum_{k \in \mathcal{N}} p_{ik}^t \phi_{ik}(j) + p_i^r \big(t_i(j) - r_i(j)\big) \Big] \tag{4}$$

We summarize above notations into the following sets:

- *input set*: $\boldsymbol{r} = \{r_i(j) \mid i,j \in \mathcal{N}\}$
- *node flow set*: $\boldsymbol{t} = \{t_i(j) \mid i,j \in \mathcal{N}\}$
- *routing variable set*: $\boldsymbol{\phi} = \{\phi_{ik}(j) \mid i,j,k \in \mathcal{N}\}$
- *power consumption set*: $\boldsymbol{p} = \{p_i \mid i \in \mathcal{N}\}$

Note that the relations among the input set and the node flow set are constrained by flow conservation. And we further have the following lemma.

Lemma 1 Given the input set $\boldsymbol{r}$ and routing variable set $\boldsymbol{\phi}$, the set of equations (1) has a unique solution for t. Each element $t_i(j)$ is nonnegative and continuously differentiable as a function of $\boldsymbol{r}$ and $\boldsymbol{\phi}$.

Detailed proofs of lemmas and theorems in this paper are postponed to the appendix.

III. Problem Formulation

Maximum network lifetime routing tries to maximally prolong the duration in which the entire network properly functions. Here we consider the network lifetime as the lifetime of the wireless node who dies first[1]. The problem of maximum network lifetime routing asks that given traffic demand, how to route the traffic so that the network lifetime can be maximized. Let T_i denote the lifetime of wireless node i, and T is the lifetime of the wireless network, $T = \min\{T_i, i \in \mathcal{N}\}$. The problem can be formulated as the following *linear optimization* problem.

$$\begin{aligned} \mathbf{T}: \quad \textbf{maximize} \quad & T \\ \textbf{subject to} \quad & p_i \cdot T \le E_i, \forall i \in \mathcal{N} \\ & \text{Eq.}(1) \text{ and Eq.}(4) \end{aligned} \tag{5}$$

In this formulation, Eq. (5) comes from the definition of lifetime, *i.e.*, the energy consumption at any node i within the network lifetime is no more than its energy reserve:

$$p_i \cdot T \le p_i \cdot T_i = E_i, \forall i \in \mathcal{N}$$

Eq. (1) and Eq. (4) represent the flow conservation and the power consumption rate. For a given network layout $(\mathcal{N}, \mathcal{L})$ and traffic input set $\boldsymbol{r}$, one can acquire the optimal routing solution $\boldsymbol{\phi}$ to maximize T by solving the above linear optimization problem via a centralized algorithm (*e.g.*[18]). The real challenge is how to solve this problem in a distributed fashion.

To address this challenge, we propose a novel utility-based nonlinear optimization formulation of the maximum lifetime routing problem, which can lead to a fully distributed routing algorithm. This formulation is inspired by the *max-min* resource allocation problem in distributed computing and networking area. *Max-min fairness* means that for any user i, increasing its resource share x_i can not be achieved without decreasing the resource share of another user x_j that satisfies $x_i \ge x_j$. Simply put, max-min allocation mechanism maximizes the resource share of the user who is allocated with the minimum resource.

In the context of maximum life time routing, if we regard lifetime of a node as a certain "resource" of its own, then the goal of maximizing network lifetime can be regarded as to "allocate lifetime" to each node so that max-min fairness criterion is satisfied. This "lifetime allocation" mechanism needs to be achieved via routing and has to satisfy the traffic input, flow conservation and power consumption constraint.

We further adopt the concept of "utility" which has been widely used in the area of resource allocation. Defined on the resource share of a user, utility usually represents the degree of satisfaction of this user. It is shown that [24], [25] by defining an appropriate utility function, the problem of achieve max-min fairness can be converted into the problem of maximizing the aggregated utility (sum of utilities of all users). Thus we

[1]We also note that there are other definitions of network lifetime, such as α-lifetime [23], which the presented formulation and algorithm can be extended.

define *utility* U_i of a node i as a function of its lifetime T_i as follows.

$$U_i(T_i) = \frac{T_i^{1-\gamma}}{1-\gamma}, \gamma \to \infty \tag{6}$$

where $T_i = E_i/p_i$ and γ can be made arbitrarily large to infinite. How to determine the value of γ and its impact will be discussed in detail in Sec. VI. It is shown that [24], [25] $T_i, i \in \mathcal{N}$ satisfy max-min fair if and only if it solves the aggregated utility maximization problem $\max \sum_{i\in\mathcal{N}} U_i$, with U_i defined as in Eq. (6).

To this end, we re-formulate the problem of maximum network lifetime as to maximize the aggregate utility of all nodes within the network.

$$\begin{aligned} \mathbf{U}: \quad & \textbf{maximize} \quad U = \sum_{i\in\mathcal{N}} U_i \\ & \textbf{subject to} \quad p_i \cdot T_i \le E_i, \forall i \in \mathcal{N} \\ & \qquad\qquad\quad \text{Eq.}(1) \text{ and Eq.}(4) \end{aligned}$$

IV. Optimality Conditions

With the presented utility-based problem formulation, we only need to seek a distributed algorithm that solves the problem **U** in order to achieve maximum lifetime routing. First we need to understand the optimality condition of such a solution. From the nonlinear optimization theory [26], we consider the first order conditions in problem **U**. Note that the utility U_i is a function of node lifetime T_i, which directly associates with p_i based on relation $T_i = E_i/p_i$. Thus we can write $U_i(p_i)$ as a function of p_i. Power consumption p_i in turn depends on the input set $\boldsymbol{r}$ and the routing variable set $\boldsymbol{\phi}$. Thus we calculate the partial derivatives of the aggregate utility U with respect to the inputs $\boldsymbol{r}$ and the routing variables $\boldsymbol{\phi}$, respectively.

We first consider $\partial U/\partial r_i(j)$, the *marginal utility* on node i with respect to commodity j. Assume that there is a small increment ϵ on the input traffic $r_i(j)$. Then $\epsilon\phi_{ik}(j)$ from this new incoming traffic will flow over wireless link (i,k). This will cause an increment power consumption $\epsilon\phi_{ik}(j)p_{ik}^t$ on node i, in order to send out the incremented traffic. And the consequent utility change of node i is

$$\epsilon\phi_{ik}(j)p_{ik}^t U_i'(p_i)$$

On the receiver side, this will cause an increment power consumption $\epsilon\phi_{ik}(j)p_k^r$ on node k, in order to receive the incremented traffic. The consequent utility change of node k is

$$\epsilon\phi_{ik}(j)p_k^r U_k'(c_k)$$

If node k is not the destination node, then the increment $\epsilon\phi_{ik}(j)$ of extra traffic at node k will cause the same utility change onward as a result of the increment $\epsilon\phi_{ik}(j)$ of input traffic at node k. To first order this utility change will be $\epsilon\phi_{ik}(j)\partial U/\partial r_k(j)$. Summing over all adjacent nodes k, then, we find that,

$$\begin{aligned} \frac{\partial U}{\partial r_i(j)} &= \sum_{k\in\mathcal{N}} \phi_{ik}(j)\Big[p_{ik}^t U_i'(p_i) + p_k^r U_k'(p_k) + \frac{\partial U}{\partial r_k(j)}\Big] \\ &= \sum_{k\in\mathcal{N}} \phi_{ik}(j)\Big[U_{ik}' + \frac{\partial U}{\partial r_k(j)}\Big] \\ &= \sum_{k\in\mathcal{N}} \phi_{ik}(j)\delta_{ik}(j) \end{aligned} \tag{7}$$

where $U_{ik}' = p_{ik}^t U_i'(p_i) + p_k^r U_k'(p_k)$ is called the *marginal utility on link* (i,k), and $\delta_{ik}(j) = U_{ik}' + \frac{\partial U}{\partial r_k(j)}$ is called the *marginal utility of link* (i,k) *with respect to commodity* j.

(7) asserts that *the marginal utility of a node is the convex sum of the marginal utilities of its outgoing links with respect to the same commodity*. By the definition of $\boldsymbol{\phi}$, we can see that $\partial U/\partial r_j(j) = 0$, since $\phi_{jk}(j) = 0$, i.e., no traffic of commodity j needs to be routed anymore once it arrives to the destination.

Next we consider $\partial U/\partial\phi_{ik}(j)$. An increment ϵ in $\phi_{ik}(j)$ causes an increment $\epsilon t_i(j)$ in the portion of $t_i(j)$ flowing on link (i,k). If $k \neq j$, this causes an addition $\epsilon t_i(j)$ to the traffic at k destined for j. Thus for $(i,k) \in \mathcal{L}$, $i \neq j$,

$$\begin{aligned} \frac{\partial U}{\partial\phi_{ik}(j)} &= t_i(j)\Big[p_{ik}^t U_i'(p_i) + p_k^r U_k'(p_k) + \frac{\partial U}{\partial r_k(j)}\Big] \\ &= t_i(j)\delta_{ik}(j) \end{aligned} \tag{8}$$

To summarize above discussions, we have the following theorem.

Theorem 1: Let a network have traffic input set $\boldsymbol{r}$ and routing variables $\boldsymbol{\phi}$, and let each marginal utility $U_i'(p_i)$ be continuous in p_i, $i \in \mathcal{N}$. Then

- the set of equations (7), $i \neq j$, has a unique solution for $\partial U/\partial r_i(j)$;
- both $\partial U/\partial r_i(j)$ and $\partial U/\partial\phi_{ik}(j)$ ($i \neq j$, $(i,k) \in \mathcal{L}$) are continuous in $\boldsymbol{r}$ and $\boldsymbol{\phi}$.

Now we proceed to show the necessary and sufficient condition for the optimal solution of maximum lifetime routing problem. Recall that the maximum network lifetime routing problem is to route the traffic so that the network lifetime can be maximized for given traffic input. Here the solution space ψ consists of all possible routing variable sets $\boldsymbol{\phi}$. The conditions for $\boldsymbol{\phi}$ to be an optimal solution of the maximum life time routing problem is given in Theorem 2.

Theorem 2: Assume that U_i is concave and continuously differentiable for $p_i, \forall i$. U is maximized if and only if for $\forall i, j\mathcal{N}$

$$\frac{\partial U}{\partial r_i(j)} \begin{cases} = \delta_{ik}(j) & \mathrm{i}f\phi_{ik} > 0 \\ \ge \delta_{ik}(j) & \mathrm{i}f\phi_{ik} = 0 \end{cases} \tag{9}$$

Theorem 2 states that the aggregate utility is maxmized if at any node i, for a given commodity j, all links (i,k) that have any portion of flow $t_i(j)$ routed through ($\phi_{ik}(j) > 0$) must achieve the same marginal utility with respect to j, and that this maximum marginal utility must be greater than or equal to the marginal utilities of the links with no flow routed ($\phi_{ik}(j) = 0$).

V. Distributed Routing

A. Overview

By understanding the optimality conditions to maximum lifetime routing, the design philosophy of our routing scheme should now be clear. The algorithm works in an iterative fashion. In each iteration, for each node i and a given commodity j, i must incrementally decrease the fraction of traffic on link (i, k) (by decreasing $\phi_{ik}(j)$) whose marginal utility $\delta_{ik}(j)$ is large, and do the reverse for those links whose marginal utility is small, until the marginal utilities of all links carrying traffic are equal. When this condition is met for all nodes and all commodities, the entire system reaches the optimal point.

Therefore, for each node i, each iteration involves two steps: (1) the calculation of marginal utility U'_{ik} for each outgoing link (i, k), and each of its downstream neighbors k's marginal utility $\partial U/\partial r_k(j)$; (2) the adjustment of routing variables $\phi_{ik}(j)$ based on the values of U'_{ik} and $\partial U/\partial r_k(j)$. We will elaborate them in details as follows.

Sec. V-B introduces how the calculation and update of marginal utilities U'_{ik} and $\partial U/\partial r_k(j)$ are executed. Sec. V-C discusses how to maintain loop-free routing. Sec. V-D formally presents the algorithm, whose optimal property is analyzed in Sec. V-E.

B. Calculation of Marginal Utilities

We first introduce how to calculate the link marginal utility $U'_{ik} = p^t_{ik}U'_i(c_i) + p^r_k U'_k(c_k)$. Sending data over a wireless link (i, k) requires power consumption of both sending node i and receiving node k. Thus the calculation of U'_{ik} depends on cooperation of both nodes. Node i is responsible to calculate the term $p^t_{ik}U'_i(p_i)$. $U'_i(p_i)$ can be derived based on Eq. (6), if the energy reserve E_i and power consumption rate p_i are known. Both values can be directly measured by node i. p^t_{ik} can be calculated based on Eq. (3), if constants α, β, m, and node distance d_{ik} are known beforehand. Alternatively, node i can directly estimate p^t_{ik} by measuring the amount of data sent from i to k and corresponding power consumption. Node k is responsible to calculate the term $p^r_k U'_k(p_k)$. $U'_k(p_k)$ can be calculated the same way as $U'_i(p_i)$. p^r_k can be either calculated based on Eq. (2), or directly estimated by measuring the amount of data received at node k and the corresponding power consumption. After calculation, k can send the value of $p^r_k U'_k(p_k)$ to node i, which in turn acquires U'_{ik}.

Now we see how each node i calculates its marginal utility $\partial U/\partial r_i(j)$, with respect to commodity j. In order to do so, based on Eq. (7), i needs to know $\delta_{ik}(j) = U'_{ik} + \partial U/\partial r_k(j)$, the marginal utility of all its outgoing links regarding commodity j. We have just discussed how to calculate U'_{ik}, and $\partial U/\partial r_k(j)$ is the marginal utility of i's downstream neighbor k. Now it is clear that $\partial U/\partial r_i(j)$ should be calculated in a recursive way. Starting from node j, the recipient of commodity j, $\partial U/\partial r_j(j) = 0$ based on definition. j then sends the values of $\partial U/\partial r_j(j)$ and $p^r_j U'_j(p_j)$ to its upstream neighbor, say k. Upon receiving the updates, node k can calculate U'_{ik} as described above, then acquire $\partial U/\partial r_k(j)$. Then, k repeats the same procedure to its upstream neighbor, until node i is reached.

C. Loop-free Routing

From the above calculation, we can see that among all nodes carrying traffic of commodity j, their marginal utilities follow a partial ordering. The recipient node of commodity j has the highest marginal utility, which is 0. Its upstream neighbors have lower marginal utilities[2], whose own upstream neighbors have even lower marginal utilities. Therefore, the recursive procedure of node marginal utility calculation is free of deadlock if and only if such a partial ordering is maintained, i.e., the routing variable set $\boldsymbol{\phi}$ is loop free.

In order to achieve loop-free routing, for each node i, with respect to commodity j, we introduce a set $B_{i,\boldsymbol{\phi}}(j)$ of blocked nodes k for which $\phi_{ik}(j) = 0$ and the algorithm is not permitted to increase $\phi_{ik}(j)$ from 0. $k \in B_{i,\boldsymbol{\phi}}(j)$ if one of the following conditions is met.

1) $(i, k) \notin \mathcal{L}$, i.e., k is not a neighbor of i.
2) $\phi_{ik} = 0$ and $\partial U/\partial r_i(j) \geq \partial U/\partial r_k(j)$, i.e., the marginal utility of k is already greater than or equal to the marginal utility of i.
3) $\phi_{ik} = 0$ and $\exists (l, m) \in \mathcal{L}$ such that (a) $l = k$ or l is downstream to k with respect to commodity j; (b) $\phi_{lm}(j) > 0$, and $\partial U/\partial r_l(j) \geq \partial U/\partial r_m(j)$, i.e., (l, m) is an improper link.

An example illustrating improper link is shown in Fig. 2. The solid line indicates that there is traffic on this link, and the dotted line indicates otherwise. Here node 4 is the destination, and all other nodes have input traffic destined to 4. The partial ordering of their marginal utilities are $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$, where the marginality of node 4 is the highest. However, the traffic from node 3 to 1 flows against such partial ordering (node 3 has higher marginal utility than node 1). Node 2, if unaware of the existence of such an improper link downstream, might make a loop by moving some of its outgoing traffic to node 3. To prevent this case from happening, node 3 only needs to raise a flag when updating its marginal utility to its upstream nodes 1 and 2. Upon receiving such a notification, nodes 1 and 2 can include node 3 into their blocking sets.

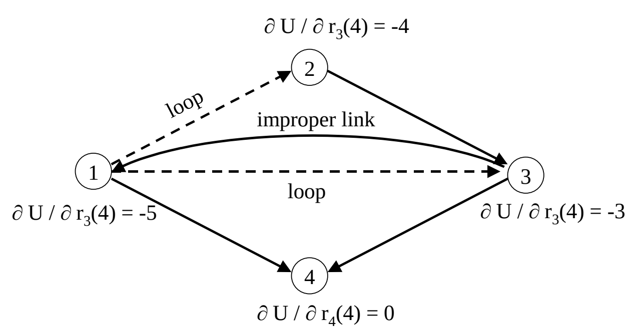


Fig. 2. Illustration of Improper Link

D. Algorithm

Now we are ready to formalize our algorithm. We use $\boldsymbol{\phi}^{(k)}$ to represent the routing variable set at the iteration k. $\Delta\boldsymbol{\phi}^{(k)}$

[2] Given the definition of marginal utility in Eq. 7, its value is non-positive

is the changes made to $\phi^{(k)}$ during the iteration k. Apparently, $\phi^{(k+1)} = \phi^{(k)} + \Delta\phi^{(k)}$. Also for node i,

- $\phi_i(j) = (\phi_{i1}(j), \ldots, \phi_{im}(j))^T$ is the vector of its routing variable regarding commodity j.
- $\Delta\phi_i(j) = (\Delta\phi_{i1}(j), \ldots, \Delta\phi_{im}(j))^T$ is the vector of changes to $\phi_i(j)$.
- $\delta_i(j) = (\delta_{i1}(j), \ldots, \delta_{im}(j))^T$ is the vector of marginal utilities of all i's neighbors.

At iteration k, node i operates according to the following steps.

1) Calculate link marginal utility U'_{ik} for each of its going links (i,k), get updates of marginal utility $\partial U/\partial r_k(j)$ from each of its downstream neighbors k, then calculate $\delta_{ik}(j) = U'_{ik} + \partial U/\partial r_k(j)$.
2) Calculate its own marginal utility $\partial U/\partial r_i(j)$ according to Eq. (7), and send it to all its upstream neighbors.
3) Calculate $\Delta\phi_i^{(k)}(j)$ as follows

$$\Delta\phi_{il}^{(k)}(j) = \left\{ \begin{array}{ll} -\min\{\phi_{il}^{(k)}(j), \frac{\rho(\delta_{il}(j)-\delta_{\min}(j))}{t_i(j)}\} & \delta_{il}(j) \neq \delta_{\min}(j) \\ \sum_{\delta_{im}(j)\neq\delta_{\min}(j)} \Delta\phi_{im}^{(k)}(j) & \delta_{il}(j) = \delta_{\min}(j) \end{array} \right. \tag{10}$$

where $\delta_{\min}(j) = \min_{m\notin B_{i,\phi^{(k)}(j)}} \delta_{im}(j)$, and $\rho > 0$ is some positive stepsize.
4) Adjust routing variables

$$\phi_i^{(k+1)}(j) = \phi_i^{(k)}(j) + \Delta\phi_i^{(k)}(j), \forall i \in \mathcal{N} - \{j\}$$

E. Analysis

The following lemma shows some of the properties of our algorithm.

Lemma 2:

(a) If $\phi^{(k)}(j)$ is loop-free, then $\phi^{(k+1)}(j)$ is loop-free.

(b) If $\phi^{(k)}(j)$ is loop-free and $\Delta\phi^{(k)}(j) = 0$ solves problem defined in step (3) of the algorithm, then $\phi^{(k)}(j)$ is optimal.

(c) If $\phi^{(k)}(j)$ is optimal, then $\phi^{(k+1)}(j)$ is also optimal.

(d) If $\Delta\phi^{(k)}(j) \neq 0$ for some i for which $t_i(j) > 0$, then

$$U(\phi^{(k)}(j) + \Delta\phi^{(k)}(j)) > U(\phi^{(k)}(j))$$

The following theorem shows the main convergence result.

Theorem 3: Let the initial routing $\phi^{(0)}$ be loop-free and satisfy $U(\phi^{(0)}) \geq U_0$ where U_0 is some scalar, then

$$U(\phi^{(k+1)}(j)) \geq U(\phi^{(k)}(j))$$
$$\lim_{k\to\infty} U(\phi^{(k+1)}(j)) = \min_{\phi(j)\in\Phi(j)} U(\phi(j))$$

Furthermore, every limit point of $\{\phi^{(k)}\}$ is an optimal solution to problem defined in step (3) of the algorithm.

VI. Experimental Studies

A. Experimental Setup

We evaluate the performance of our routing algorithm via simulation in this section. Our simulation setting is as follows. We randomly create 100 nodes on a $100 \times 100m^2$ square. The maximum transmission range of each node is $25m$. In our experiment, we set $\alpha = 50nJ/b$, $\beta = 0.0013pJ/b/m^4$ and $m = 4$ for the power consumption model. The energy reserve on each node is $50kJ$.

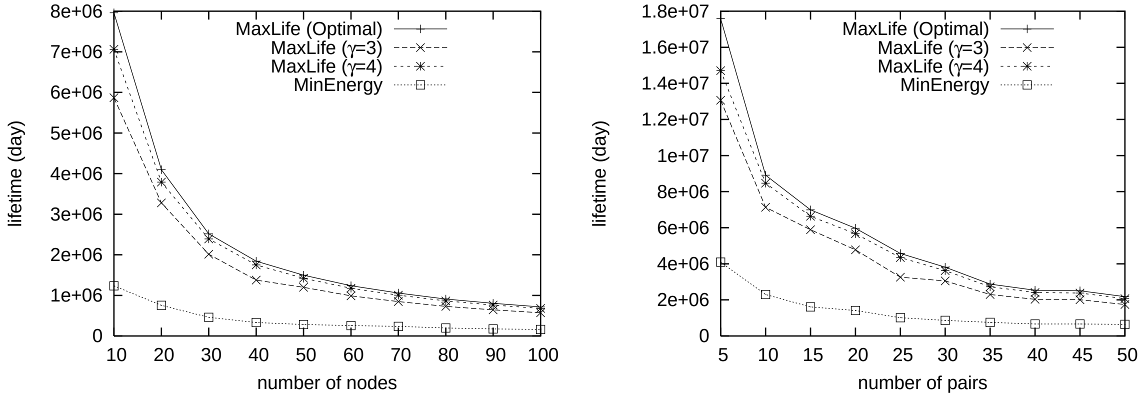
We study two types of networking scenarios, *sensor network* and *ad hoc network*. In the scenario of sensor network, a node is picked as the base station (data sink), while a subset of other nodes act as data sources sending traffic to the sink at $0.5Kbps$. The rest of the nodes act as relaying nodes. In the scenario of *ad hoc network*, we randomly create several pairs of unicast connection. Besides the senders and receivers of these unicast pairs, other nodes are responsible to relay traffic. The sending rate of each connection is $0.5Kbps$. We run each experiment over 20 different random topologies. For example, when evaluating the network lifetime of the sensor network scenario with 40 sensors, we create 20 different 100-node topologies and pick 40 nodes as data sources. We run algorithms on each of them, then show the average result.

We compare the performance of following algorithms. *MinEnergy* algorithm tries to minimize the energy consumption for each data unit routed through the network. For each data source, the algorithm finds its shortest path to the destination in terms of energy cost. The route for each data source is fixed throughout the entire network lifetime. *MaxLife* algorithm tries to maximize the network lifetime, which can be implemented in centralized or distributed fashions. The centralized algorithm derives the maximum lifetime by solving the linear programming problem (**T**). The distributed algorithm is the one presented in Sec. V. As mentioned in Sec. III, our distributed algorithm can only converge to near-optimal routing unless $\gamma \to \infty$. So we also evaluate the performance of our algorithm when γ takes different values.

B. Network Lifetime

Fig. 3 (a) shows the lifetime of the same sensor network when the data is routed by different algorithms under the single-sink setting. We observe that the maximum lifetime of the network drops as a super linear speed as the number of data sources increases, mainly as a result of increased traffic demand in a network with fixed overall energy reserve. It also shows the optimal (centralized) *MaxLife* algorithm consistently maintains the network lifetime at least 5 times the same result returned by the *MinEnergy* algorithm. Also the performance of our distributed algorithm is able to approximate performance of the optimal algorithm by an average 80% when $\gamma = 3$. When $\gamma = 4$, the average approximation ratio increases to 95%.

From Fig. 3 (b), we have the same observations except that in the scenario of ad hoc network, our algorithm outperforms the *MinEnergy* algorithm by only 3 times. This is mainly due to the different traffic patterns of two networking scenarios. In the sensor network scenario, the *MinEnergy* algorithm ends up with a shortest-path data aggregation tree, where the entire traffic concentrate on a few nodes locating close to the data sink. The energy reserve of these nodes can easily run out soon, which is the main reason for the inferior performance of this algorithm. The same traffic concentration problem exists in ad hoc network scenario, but not as serious as the previous case. In contrast, in both networking scenarios, our algorithm



(a) Sensor Network (b) Ad-hoc Network

Fig. 3. Network Lifetime

is able to effectively diverge the traffic, hence the energy consumption, among all nodes, which significantly prolongs the network lifetime.

C. Energy Cost

On the other hand, our algorithm consumes more system energy than *MinEnergy* algorithm for an average bit of data routed through the network. The reason is that, in order to maximally utilize the energy reserve of all nodes within the network, sometimes the data from a source has to go through some route whose energy consumption rate is not as efficient as the one returned by *MinEnergy* algorithm. As shown in Fig. 4, such an inefficiency is bounded by a factor of 2 under various experimental settings.

D. Distribution of Energy Consumption

The distinction of two algorithms' energy consumption pattern is further exhibited in Fig. 5, which plots the distribution of the energy consumption ratio of each node in the network throughout the entire lifetime. In both networking scenarios, 50 out of 100 nodes are send traffic. For *MinEnergy* algorithm, the distribution is highly asymmetric. Under both networking scenarios, only a few "hot spot" nodes completely utilize their energy reserves, while about 40% of the nodes do not consume any energy at all, since they are not included in the unicast routes or data aggregation tree.

On the other hand, in the result of our algorithm, most nodes get to contribute about 60% of its energy reserve, since in our algorithm, each node always tend to allocate more traffic via the with the maximum marginal utility, i.e., the node with the least power consumption ratio. Thus, our algorithm is able to sustain a much longer system lifetime at the price of more energy consumption per bit than the *MinEnergy* algorithm.

VII. Related Works

Besides the work on energy-efficient routing algorithms as we have discussed in Sec. I, there are other research efforts to address the constrained-energy problem in wireless networks.

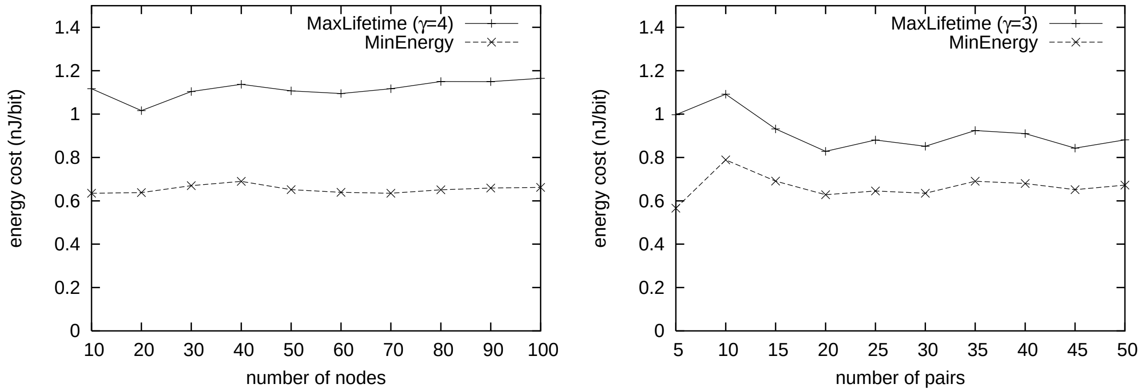
Some works explore the performance limits of energy-constrained wireless networks. In particular, Hu and Li [27] study the the energy-constrained fundamental limites with respect to the network throughput and lifetime in wireless sensor networks. They give asymptotic analytical results on the relationship between network lifetime and the number of nodes with fixed node density. Zhang and Hou [23] derive the necessary and sufficient condition of the node density in order to maintain k-coverage and the upper bound of network lifetime when only α-portion of a region is required to be covered, given a fixed node density in this finite region. Our work is different from these works in that we study *how to* achieve the upper bound of network lifetime via distributed routing, instead of exploring *what is* this upper bound.

Hou et al. [28] study the problem of rate allocation with the requirement of network lifetime. The work of [29] also presents a rate allocation algorithm based on traffic splits in ad hoc networks. Our algorithm addresses a different problem from these works and may fit in different network operation environments. In particular, in our problem, the traffic demands from all sensors are fixed and known a priori. This problem well models the application scenarios, such as temperature, pressure, noise level monitoring, where fixed amount of information is generated at a fixed interval. Our goal is to maximize the network lifetime while satisfying the rate demands, instead of allocating rates to different wireless nodes such that certain fairness criteria are satisfied.

The presented distributed routing algorithm in this paper is similar to the works of [21], [22] in that both algorithms explore the *marginal* utility (delay in their cases) to achieve the optimum in a distributed way. Yet this work studies a different network problem than these previous works. First, the goal of this work is to maximize the network lifetime, while the works of [21], [22] are to minimize the aggregate delay of the network. Second, this work studies the wireless nodes with energy constraint, while they study the wireline links which incurs delay when overloaded.

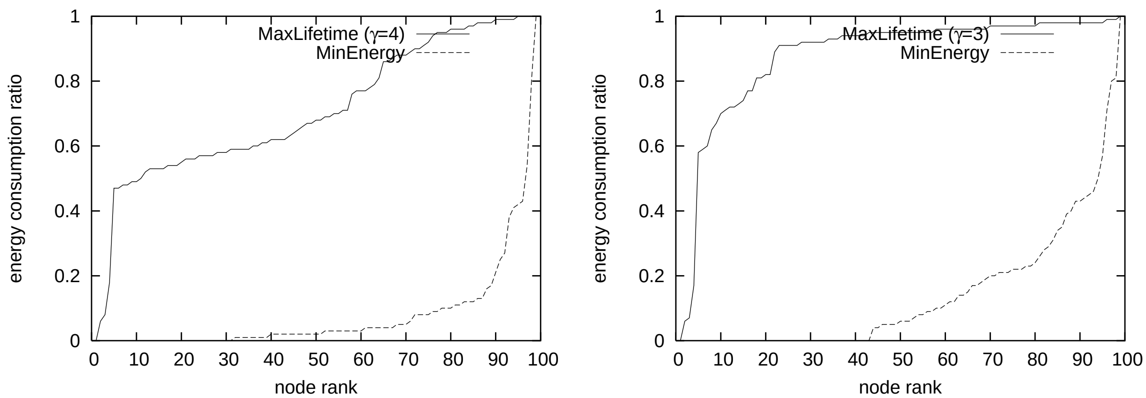
VIII. Conclusion

This paper studies the problem of distributed maximum network lifetime routing for multihop wireless network. Inspired by the max-min fair resource allocation, this paper presents a novel utility-based nonlinear formulation of this



(a) Sensor Network (b) Ad-hoc Network

Fig. 4. Average Energy Cost



(a) Sensor Network (50 Data Sources) (b) Ad-hoc Network (25 Unicast Pairs)

Fig. 5. Energy Consumption Distribution

problem. Based on this formulation, this paper further presents a distributed routing algorithm that achieves the goal of maximizing network lifetime. The presented algorithm has solid non-linear optimization theoretical background and is shown to be effective and efficient under various simulation environments.

Appendix A

Proof of Lemma 1 *Proof:* Without loss of generality, let us consider the commodity j. We restate Eq. (1) as

$$t_i(j) = r_i(j) + \sum_{l \in \mathcal{N}-\{j\}} t_l(j)\phi_{li}(j), \forall i \in \mathcal{N} \tag{11}$$

since $\phi_{lj}(j) = 0$. Summing both sides over i, we have

$$t_j(j) = \sum_{i \in \mathcal{N}} r_i(j) \tag{12}$$

The physical meaning of (12) is obvious: the amount of commodity arrived at node j equals the total amount generated from each of its sources. For the pure purpose of proof, we temporarily define $\phi_{ji}(j) = r_i(j)/t_j(j)$ and substitute it into (11), we have

$$t_i(j) = \sum_{l \in \mathcal{N}} t_l(j)\phi_{li}(j), i \in \mathcal{N} \tag{13}$$

Any solution to (13) and (12) satisfies (11). Let $\hat{\mathbf{\Phi}}(j)$ be the $n \times n$ matrix with elements $\phi_{li}(j)$. $\hat{\mathbf{\Phi}}(j)$ is stochastic, since each element $\phi_{li}(j) \geq 0$, and $\sum_{i=1}^{n} \phi_{li}(j) = 1$ $(1 \leq i \leq n)$. Consequently, (13) is the formula for steady-state probabilities in a Markov chain.

If $\hat{\mathbf{\Phi}}(j)$ is irreducible, then (13) has a unique solution. In order to make $\hat{\mathbf{\Phi}}(j)$ irreducible, there has to exist a path between any pair i and k, i.e., $\phi_{il}(j) > 0, \phi_{lm}(j) > 0, \ldots, \phi_{pk}(j) > 0$. To prove this, we only need to show that there exists a path from node j to any other node, and a path from any other node to j. For a node i, if $r_i(j) > 0$, then there is a path from i to j. Otherwise, the traffic generated from i will not arrive at j, contradicting (12). Also by the temporary definition of $\phi_{ji}(j) = r_i(j)/t_j(j)$, there is a path from j to i too. In conclusion, if $r_i(j) > 0 (i \in \mathcal{N} - \{j\})$, then $\hat{\mathbf{\Phi}}(j)$ is irreducible, hence (13) has a unique solution, where $t_i(j) > 0 (i \in \mathcal{N} - \{j\})$.

If we remove the jth column and jth row of $\hat{\mathbf{\Phi}}(j)$, we acquire a $(n-1) \times (n-1)$ matrix $\mathbf{\Phi}(j)$. If we define two row vectors as:

$$\begin{aligned} \boldsymbol{t}(j) &= (t_1(j), \ldots, t_{j-1}(j), t_{j+1}(j), \ldots, t_n(j)) \\ \boldsymbol{r}(j) &= (r_1(j), \ldots, r_{j-1}(j), r_{j+1}(j), \ldots, r_n(j)) \end{aligned}$$

then we can restate (11) into the following vector form:

$$\boldsymbol{t}(j)(\boldsymbol{I} - \boldsymbol{\Phi}(j)) = \boldsymbol{r}(j)$$

Since this equation has a unique solution if $\boldsymbol{r}(j) > 0$, $I - \boldsymbol{\Phi}(j)$ must have an inverse. Therefore,

$$\boldsymbol{t}(j) = \boldsymbol{r}(j)(\boldsymbol{I} - \boldsymbol{\Phi}(j))^{-1} \tag{14}$$

Since $\boldsymbol{t}(j)$ is positive when $\boldsymbol{r}(j)$ is positive, $\boldsymbol{t}(j)$ is nonnegative when $\boldsymbol{r}(j)$ is nonnegative. Now we differentiate $\boldsymbol{t}(j)$ as a function of $\boldsymbol{r}(j)$. Differentiating (14), we get the continuous function of $\boldsymbol{\Phi}(j)$,

$$\frac{\partial t_i(j)}{\partial r_l(j)} = [(I - \boldsymbol{\Phi}(j))^{-1}]_{li} \tag{15}$$

Using (15) in (14), we can express the solution to (11) as

$$t_i(j) = \sum_{l \in \mathcal{N} - \{j\}} \frac{\partial t_i(j)}{\partial r_l(j)} r_l(j) \tag{16}$$

Now we differentiate $\boldsymbol{t}(j)$ as a function of $\boldsymbol{\Phi}(j)$. Differentiating (11) with $\phi_{km}(j)$, we get

$$\frac{\partial t_i(j)}{\partial \phi_{km}(j)} = \begin{cases} \sum_{l \in \mathcal{N} - \{j\}} \frac{\partial t_l(j)}{\partial \phi_{km}(j)} \phi_{li}(j) + t_k(j) & \text{if } i = m \\ \sum_{l \in \mathcal{N} - \{j\}} \frac{\partial t_l(j)}{\partial \phi_{km}(j)} \phi_{li}(j) & \text{otherwise} \end{cases} \tag{17}$$

If we fix k and m, and introduce two variables α_i and β_i defined as

$$\begin{aligned} \alpha_i(j) &= \frac{\partial t_i(j)}{\partial \phi_{km}(j)} \\ \beta_i(j) &= \begin{cases} t_k & \text{if } i = m \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

(17) becomes

$$\alpha_i(j) = \beta_i(j) + \sum_{l \in \mathcal{N} - \{j\}} \alpha_i(j) \phi_{li}(j), i \in \mathcal{N}$$

which has the same set of equations as (11), with $\alpha_i(j)$ corresponding to $t_i(j)$, and $\beta_i(j)$ corresponding to $r_i(j)$. Also since $\beta_i(j) \geq 0$, we can repeat the same derivation for $t_i(j)$ and $r_i(j)$ and reach the same conclusion as in (15) and (16):

$$\frac{\partial \alpha_i(j)}{\partial \beta_l(j)} = \frac{\partial t_i(j)}{\partial r_l(j)} = [(\boldsymbol{I} - \boldsymbol{\Phi}(j))^{-1}]_{li}$$

$$\alpha_i(j) = \sum_{l \in \mathcal{N}} \frac{\partial \alpha_i(j)}{\partial \beta_l(j)} \beta_l(j) = \frac{\partial \alpha_i(j)}{\partial \beta_m(j)} \beta_m(j)$$

Substituting $\frac{\partial t_i(j)}{\partial \phi_{km}(j)}$ and $t_k(j)$ back to the above equation, we have the solution, continuous in $\boldsymbol{\phi}(j)$, as

$$\frac{\partial t_i(j)}{\partial \phi_{km}(j)} = \frac{\partial t_i(j)}{\partial r_m(j)} t_k(j) \tag{18}$$

■

APPENDIX B

Proof of Theorem 1 *Proof:* Without loss of generality, let us consider the commodity j. Let $b_i(j) = \sum_{k \in \mathcal{N}} \phi_{ik}(j) U'_{ik}$. We define two column vectors as:

$$\begin{aligned} \boldsymbol{b}(j) &= (b_1(j), \ldots, b_{j-1}(j), b_{j+1}(j), \ldots, b_n(j))^T \\ \boldsymbol{\nabla} \cdot U(j) &= (\frac{\partial U}{\partial r_1(j)}, \ldots, \frac{\partial U}{\partial r_{j-1}(j)}, \frac{\partial U}{\partial r_{j+1}(j)}, \ldots, \frac{\partial U}{\partial r_n(j)})^T \end{aligned}$$

then we can rewrite (7) into the following vector form:

$$\boldsymbol{\nabla} \cdot U(j) = \boldsymbol{b} + \boldsymbol{\Phi}(\boldsymbol{\nabla} \cdot U) \tag{19}$$

We saw in the proof of Theorem 1 that $\boldsymbol{I} - \boldsymbol{\Phi}$ has a unique inverse with elements given by (15). Thus the unique solution to (19), continuous in $\boldsymbol{\Phi}(j)$, is given by

$$\boldsymbol{\nabla} \cdot U(j) = (\boldsymbol{I} - \boldsymbol{\Phi})^{-1} \boldsymbol{b}$$

Substituting $\sum_{k \in \mathcal{N}} \phi_{ik}(j) U'_{ik}$ back to the above equation, we have

$$\frac{\partial U}{\partial r_i(j)} = \sum_{l \in \mathcal{N}} \frac{\partial t_l(j)}{\partial r_i(j)} \sum_{m \in \mathcal{N}} \phi_{lm}(j) U'_{lm} \tag{20}$$

Finally, differentiating U with $\phi_{ik}(j)$ using (1) and (4), we have

$$\frac{\partial D}{\partial \phi_{ik}(j)} = \sum_{(l,m) \in \mathcal{L}} \Big[U'_{lm} \phi_{lm}(j) \frac{\partial t_l(j)}{\partial \phi_{ik}(j)} + U'_{ik} t_i(j) \Big]$$

By (18), we have

$$\frac{\partial U}{\partial \phi_{ik}(j)} = t_i(j) \sum_{(l,m) \in \mathcal{L}} \Big[U'_{lm} \phi_{lm}(j) \frac{\partial t_l(j)}{\partial r_k(j)} + t_i(j) U'_{ik} \Big]$$

By (20), we have

$$\frac{\partial U}{\partial \phi_{ik}(j)} = t_i(j) \Big[\frac{\partial U}{\partial r_k(j)} + U'_{ik} \Big]$$

which is the same as (8). Now we can conclude that (8) is continuous in $\boldsymbol{\phi}(j)$ given the continuity of $t_i(j)$ and $\frac{\partial U}{\partial r_i(j)}$. ■

APPENDIX C

Proof of Theorem 2 *Proof:* First we show that the necessary condition to maximize U is

$$\frac{\partial U}{\partial \phi_{ik}(j)} = \begin{cases} = \max_l \partial U / \partial \phi_{il}(j) & \text{if } \phi_{ik} > 0 \\ \leq \max_l \partial U / \partial \phi_{il}(j) & \text{if } \phi_{ik} = 0 \end{cases} \tag{21}$$

We first assume that $\boldsymbol{\phi}$ does not satisfy (21). This means that there is some i,j,k, and m such that

$$\phi_{ik}(j) > 0, \frac{\partial U(\boldsymbol{\phi})}{\partial \phi_{ik}(j)} < \frac{\partial U(\boldsymbol{\phi})}{\partial \phi_{im}(j)}$$

Since these derivatives are continuous, a sufficiently small increase in $\phi_{im}(j)$ and corresponding decrease in $\phi_{ik}(j)$ will increase U, contradicting the fact that $\boldsymbol{\phi}$ maximizes U.

Next we show that the sufficient condition to maximize U is

$$\delta_{ik}(j) \leq \frac{\partial U}{\partial r_i(j)}, \forall i \neq j, (i,k) \in \mathcal{L} \tag{22}$$

Suppose that ϕ satisfies (22) and has power consumption set $\boldsymbol{c}$. Let $\boldsymbol{\phi}^*$ be any other set of routing variables with power consumption set $\boldsymbol{p}^*$. Define

$$p_i(\lambda) = \lambda p_i^* + (1-\lambda)p_i \quad (23)$$

$$U(\lambda) = \sum_{i\in\mathcal{N}} U_i(c_i(\lambda)) \quad (24)$$

Since each utility function U_i is a concave function of the power consumption $\boldsymbol{p}$, $U(\lambda)$ is convex in λ, and hence

$$\frac{dU(\lambda)}{d\lambda}\Big|_{\lambda=0} \geq U(\boldsymbol{\phi}^*) - U(\boldsymbol{\phi})$$

Since $\boldsymbol{\phi}^*$ is arbitrary, proving that $dU(\lambda)/d\lambda \leq 0$ at $\lambda = 0$ will complete the proof. From (24) to (23),

$$\frac{dU(\lambda)}{d\lambda}\Big|_{\lambda=0} = \sum_{i\in\mathcal{N}} U_i'(p_i)(p_i^* - p_i) \quad (25)$$

We now show that

$$\sum_{i\in\mathcal{N}} U_i'(p_i)p_i^* \leq \sum_{(j,k)\in\mathcal{L}} r_k(j)\frac{\partial U(\boldsymbol{\phi})}{\partial r_k(j)} \quad (26)$$

Note from (22) that

$$U_{ik}' \leq \frac{\partial U(\boldsymbol{\phi})}{\partial r_i(j)} - \frac{\partial U(\boldsymbol{\phi})}{\partial r_k(j)}$$

Multiplying both sides of the inequality by $\phi_{ik}^*(j)$, and summing over k, we have

$$\sum_{k\in\mathcal{N}} \phi_{ik}^*(j)U_{ik}' \leq \frac{\partial U(\boldsymbol{\phi})}{\partial r_i(j)} - \sum_{k\in\mathcal{N}} \frac{\partial U(\boldsymbol{\phi})}{\partial r_k(j)}\phi_{ik}^*(j) \quad (27)$$

Multiplying both sides of (27) by $t_i^*(j)$, and summing over i, j, we have

$$\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}} t_i^*(j)\sum_{k\in\mathcal{N}} \phi_{ik}^*(j)U_{ik}' \leq \sum_{i,j\in\mathcal{N}} t_i^*(j)\frac{\partial U(\boldsymbol{\phi})}{\partial r_i(j)} - \sum_{i,j,k\in\mathcal{N}} t_i^*(j)\phi_{ik}^*(j)\frac{\partial U(\boldsymbol{\phi})}{\partial r_k(j)} \quad (28)$$

From (1), $\sum_{i\in\mathcal{N}} t_i^*(j)\phi_{ik}^*(j) = t_k^*(j) - r_k(j)$. Substituting this into the left side of (28), it becomes

$$\sum_{i\in\mathcal{N}} U_i'(p_i)\sum_{j\in\mathcal{N}} t_i^*(j)\sum_{k\in\mathcal{N}} \phi_{ik}^*(j)p_{ik}^t + \sum_{k\in\mathcal{N}} U_k'(p_k)\sum_{j\in\mathcal{N}} p_k^r(t_k^*(j) - r_k(j)) \quad (29)$$
$$= \sum_{i\in\mathcal{N}} U_i'(p_i)p_i^*$$

if we replace k with i in item (29). Making the same substitution at the right side of (28), ((28)) becomes (26). Note that if we replace $\boldsymbol{\phi}^*$ with $\boldsymbol{\phi}$ in (27), (27) becomes an equality from the equation for $\partial U/\partial r_i(j)$ in (7). For the same reason, if we replace $\boldsymbol{\phi}^*$ with $\boldsymbol{\phi}$ in (28), (28) becomes

$$\sum_{i\in\mathcal{N}} U_i'(p_i)p_i = \sum_{j,k} r_k(j)\frac{\partial U(\boldsymbol{\phi})}{\partial r_k(j)} \quad (30)$$

Substituting (30) and (26) into (25), we see that $dU(\lambda)/d\lambda \leq 0$ at $\lambda = 0$, which completes the proof for the sufficient condition (22).

Combining the necessary condition (21) and sufficient condition (22), we have (9), which completes the proof. ■

We omit the proof to **Lemma 2** and **Theorem 3** due to page limit.

Appendix D

Proof of Lemma 2 *Proof:* (a) Assume that $\boldsymbol{\phi}^{(k+1)}(j)$ is not loop-free so that there exists a sequence of overlay links forming a directed cycle along which $\boldsymbol{\phi}^{(k+1)}(j)$ is positive. for which $\frac{\partial U(\boldsymbol{\phi}^{(k)}(j))}{\partial r_m(j)} \geq \frac{\partial U(\boldsymbol{\phi}^{(k)}(j))}{\partial r_n(j)}$. From the definition of $B_{m,\boldsymbol{\phi}^{(k)}}(j)$ we must have $\phi_{mn}^{(k)}(j) > 0$ and hence (m,n) is an improper link. Now move backwards around the cycle to the first link (i,l) for which $\phi_{il}^{(k)}(j) = 0$. Such a link must exist since $\boldsymbol{\phi}^{(k)}(j)$ is loop-free. Since node l is upstream of node m and link (m,n) is improper, we have $l \in B_{i,\boldsymbol{\phi}^{(k)}}(j)$ which contradicts the hypothesis $\phi_{il}^{(k+1)}(j) > 0$.

(b) If $\Delta\boldsymbol{\phi}^{(k)}(j) = 0$ solves problem defined in step (3) of the algorithm in Sec. V-D, then we must have $\boldsymbol{\delta}_i(j)^T\Delta\boldsymbol{\phi}_i(j) \leq 0$ for each node i and $\Delta\boldsymbol{\phi}(j)$ satisfying its constraint.

$$\Delta\boldsymbol{\phi}_i(j) \leq -\boldsymbol{\phi}_i^{(k)}(j), \sum_{l\in\mathcal{N}} \Delta\phi_{il}(j) = 0, \forall l \in B_{i,\boldsymbol{\phi}^{(k)}}(j)$$

By writing $\Delta\boldsymbol{\phi}_i(j) = \boldsymbol{\phi}_i(j) - \boldsymbol{\phi}_i^{(k)}(j)$ and using (7), (8) we have

$$\begin{aligned}\boldsymbol{\delta}_i(j)^T(\boldsymbol{\phi}_i(j) - \boldsymbol{\phi}_i^{(k)}(j)) &= \sum_{l\in\mathcal{N}} \delta_{il}(j)\phi_{il}(j) - \sum_{l\in\mathcal{N}} \delta_{il}(j)\phi_{il}^{(k)}(j) \\ &= \sum_{l\in\mathcal{N}} \delta_{il}(j)\phi_{il}(j) - \frac{\partial U}{\partial r_i(j)} \leq 0\end{aligned}$$

By considering $\phi_{il}(j) = 1$ for each $l \notin B_{i,\boldsymbol{\phi}^{(k)}}(j)$, we obtain

$$\frac{\partial U}{\partial r_i(j)} \geq \delta_{il}(j), \forall l \notin B_{i,\boldsymbol{\phi}^{(k)}}(j)$$

From (7) and (8) we have

$$\frac{\partial U}{\partial r_i(j)} = \delta_{il}(j), \forall l \notin B_{i,\boldsymbol{\phi}^{(k)}}(j), \phi_{il}^{(k)} > 0$$

Since $U_{il}' > 0$ for all $(i,l) \in \mathcal{L}$ it follows from (7), (8) and the relation above that there are not improper links, and using the definition of $B(i;\boldsymbol{\phi}^{(k)})$ we obtain

$$\frac{\partial U}{\partial r_i(j)} = \max_{l\in\mathcal{N}} \delta_{il}(j)$$

which is the same as (22), the sufficient condition for optimality of $\boldsymbol{\phi}^{(k)}(j)$.

(c) If $\boldsymbol{\phi}^{(k)}$ is optimal then from the necessary condition for optimality (21) we have that for all node i with $t_i(j) > 0$

$$\frac{\partial U}{\partial r_i(j)} = \max_{m\in\mathcal{N}} \delta_{im}(j)$$

It follows using a reverse argument to the one in (b) that $\Delta\boldsymbol{\phi}_i^{(k)}(j) = 0$ if $t_i(j) > 0$. Since changing only routing variables of nodes i for which $t_i(j) = 0$ does not affect the flow through each link we have $U(\boldsymbol{\phi}^{(k)}) = U(\boldsymbol{\phi}^{(k+1)})$ and

$\boldsymbol{\phi}^{(k+1)}$ is optimal.

(d) If $t_i(j) > 0$, then $\boldsymbol{M}_i^{(k)}(j)$ is positive definite on the appropriate subspace. If in addition $\Delta\boldsymbol{\phi}_i^{(k)}(j) \neq 0$, then $-\frac{t_i(j)}{2\alpha}\Delta\boldsymbol{\phi}_i(j)^T\boldsymbol{M}_i^{(k)}(j)\Delta\boldsymbol{\phi}_i(j)$ is negative. Since the maximum of the entire equation is non-negative, $\boldsymbol{\delta}_i(j)^T\Delta\phi_i^{(k)}(j) > 0$. By (8), we obtain that

$$\left(\frac{\partial U}{\partial\boldsymbol{\phi}_i(j)}\right)^T\Delta\boldsymbol{\phi}_i^{(k)}(j) > 0$$

Hence $\Delta\boldsymbol{\phi}^k(j)$ is a direction of ascent at $\boldsymbol{\phi}^k(j)$ and the result follows. ■

References

[1] A. Sankar and Z. Liu, "Maximum Lifetime Routing in Wireless Ad-hoc Networks," in *Proc. of INFOCOM*, 2004.

[2] J. Chang and L. Tassiulas, "Energy Conserving Routing in Wireless Ad-hoc Networks," in *Proc. of INFOCOM*, 2000, pp. 22–31.

[3] J. Chang and L. Tassiulas, "Fast Approximate Algorithms for Maximum Lifetime Routing in Wireless Ad-hoc Networks," in *Proc. of NETWORKING*, 2000, pp. 702–713.

[4] S. Singh, M. Woo and C. Raghavendra, "Power-aware Routing in Mobile Ad-hoc Networks," in *Proc. of MOBICOM*, 1998, pp. 181–190.

[5] K. Kar, M. Kodialam, T. V. Lakshman and L. Tassiulas, "Routing for Network Capacity Maximization in Energy-constrained Ad-hoc Networks," in *Proc. of INFOCOM*, 2003.

[6] K. Dasgupta K. Kalpakis and P. Namjoshi, "Efficient Algorithms for Maximum Lifetime Data Gathering and Aggregation in Wireless Sensor Networks," *Computer Networks*, vol. 42, pp. 697–716, 2003.

[7] S. Lindsey, C. S. Raghavendra and K. Sivalingam, "Data Gathering Algorithms in Sensor Networks Using Energy Metrics," *IEEE Tran. on Parallel and Distributed Systems*, 2002.

[8] M. Bhardwaj and A.P. Chandrakasan, "Bounding the Lifetime of Sensor Networks Via Optimal Role Assignments," in *Proc. of INFOCOM*, 2002, pp. 1587–1596.

[9] K. Dasgupta, M. Kukreja and K. Kalpakis, "Topology-Aware Placement and Role Assignment for Energy-Efficient Information Gathering in Sensor Networks," in *Proc. of the 8th IEEE Symposium on Computers and Communications*, 2003.

[10] X. Cheng, B. Narahari, R. Simha, M. X. Cheng, and D. Liu, "Strong Minimum Energy Topology in Wireless Sensor Networks: NP-Completeness and Heuristics," *IEEE Tran. on Mobile Computing*, vol. 2, no. 3, 2003.

[11] R. Wattenhofer, L. Li, P. Bahl and Y. Wang, "Distributed Topology Control for Power Efficient Operation in Multihop Wireless Ad Hoc Networks," in *Proc. of INFOCOM*, 2001.

[12] I. Stojmenovic and X. Lin, "Power-aware Localized Routing in Wireless Networks," *IEEE Tran. on Parallel and Distributed Systems*, vol. 12, no. 11, pp. 1122–1133, 2001.

[13] Y. Xue and B. Li, "Location-Aided Power-Aware Routing for Wireless Ad-hoc Networks," in *Proc. of Globecom*, 2001.

[14] J. Gomez, A. Campbell, M. Naghshineh and C. Bisdikian, "Conserving Transmission Power in Wireless Ad Hoc Networks," in *Proc. of 9th International Conference on Network Protocols (ICNP)*, 2001.

[15] V. Rodoplu and T. Meng, "Minimum Energy Mobile Wireless Networks," *IEEE Journal of Selected Areas in Communications*, vol. 17, no. 8, pp. 1333–1344, 1999.

[16] G. Zussman and A. Segall, "Energy Efficient Routing in Ad Hoc Disaster Recovery Networks," in *Proc. of INFOCOM*, 2003.

[17] Q. Li, J. Aslam and D. Rus, "Online power-aware routing in wireless Ad-hoc networks," in *Proc. of MOBICOM*, 2001, pp. 97–107.

[18] Naveen Garg and Jochen Konemann, "Faster and Simpler Algorithms for Multicommodity Flow and Other Fractional Packing Problems," in *Proc. of IEEE Symposium on Found. of Comp. Science*, 1998, pp. 300–309.

[19] B. Awerbuch and T. Leighton, "A simple local-control approximation algorithm for multicommodity flow," in *Proc. of 3rd IEEE Symposium on Found. of Comp. Science*, 1993, pp. 459–468.

[20] B. Awerbuch and T. Leighton, "Improved approximation algorithms for the multi-commodity flow problem and local competitive routing in dynamic networks," in *Proc. of the 26th annual ACM symposium on theory of computing*, 1994, pp. 487–496.

[21] D.P. Bertsekas, E.M. Gafni and R.G. Gallager, "Second derivative algorithms for minimum delay distributed routing in networks," *IEEE Tran. on Communications*, vol. 32, no. 8, 1984.

[22] R.G. Gallager, "A minimum delay routing algorithm using distributed computation," *IEEE Tran. on Communications*, vol. 25, no. 1, 1977.

[23] H. Zhang and C. Hou, "On deriving the upper bound of α-lifetime for large sensor networks," in *Proc. of ACM Mobihoc*, 2004.

[24] F. P. Kelly, "Charging and Rate Control for Elastic Traffic," *European Trans. on Telecommunications*, vol. 8, pp. 33–37, 1997.

[25] R. Srikant, *The Mathematics of Internet Congestion Control*, Birkhauser, 2004.

[26] D. Bertsekas, *Nonlinear Programming*, Athena Scientific, 1999.

[27] Z. Hu and B. Li, "On the Fundamental Capacity and Lifetime Limits of Energy-Constrained Wireless Sensor Networks," in *The 10th IEEE Real-Time and Embedded Technology and Applications Symposium (RTAS 2004)*, 2004.

[28] T. Hou, Y Shi, and H. Sherali, "Rate Allocation in Wireless Sensor Networks with Network Lifetime Requirement," in *Proc. of ACM Mobihoc*, 2004.

[29] V. Srinivasan, C.F. Chiasserini, P. Nuggehalli and R. R. Rao, "Optimal Rate Allocation and Traffic Splits for Energy Efficient Routing in Ad Hoc Networks," in *Proc. of INFOCOM*, 2002.